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Mixed numerical–experimental identification of elastic properties of orthotropic metal plates

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Abstract

This paper compares results of three different methods to determine the in-plane elastic properties of sheet materials. Results obtained with standard resonant beam and tensile tests are used to assess a mixed numerical–experimental technique developed to determine the in-plane elastic properties of orthotropic plates from the resonance frequencies of rectangular plate samples (the so-called 'Resonalyser' technique). Test materials were selected on the basis of an expected low degree of elastic anisotropy in order to put the accuracy and sensitivity of the different techniques to assess anisotropic materials to a test. Therefore, aluminium alloy and stainless steel samples were prepared from hot-rolled plates, deliberately avoiding pronounced cold-rolling textures. The differences between the results obtained with the three experimental approaches are critically evaluated.

In the case of very thin plates, the existing mixed numerical–experimental Resonalyser procedure succeeded in accurately identifying the elastic material properties. A slightly adapted procedure is proposed to extend the applicability of the Resonalyser procedure to thicker plates.

Keywords: Material identification; Elastic properties; Vibration testing methods

1. Introduction

Many engineering materials behave in an anisotropic manner: their response to external solicitations depends on the loading direction. A simple but common form of anisotropy is orthotropy. The general stress–strain relation for materials having orthotropic symmetry properties is given by Eq. (1)

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \quad (1)$$

In this expression, $\{\varepsilon_i\}$ and $\{\gamma_i\}$ represent the normal and shear strain components respectively, $\{\sigma_j\}$ and $\{\tau_j\}$ the normal and shear stress components, E_i the Young's modulus in the i-direction, ν_{ij} and G_{ij} are the Poisson's ratio and the shear modulus in the (i,j)-plane. When the assumption of the Kirchhoff plate theory [1] are respected, the material is in a state of plane stress: $\sigma_3 = \tau_{23} = \tau_{13} = 0$. In this situation, the three-dimensional stress–strain relation of Eq. (1) is reduced to a two-dimensional relation. The elastic behaviour of orthotropic materials like rolled metal sheets or long-fibre reinforced composites can thus be described by the following reduced stress–strain relation:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (2)$$

If linear material behaviour is assumed, the elastic properties E_i, ν_{ij} and G_{12} are also called the 'engineering constants'.

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Due to the symmetry of the compliance matrix in Eq. (2), only four (instead of five) independent engineering constants occur: e.g. E_1, E_2, ν_{12} and G_{12}.

Knowledge of the elastic properties of materials is important for their use in structural applications, as well as for the improvement of the processes used to transform the materials into components. Elastic properties also play a major role in the vibration behaviour of constructions. This observation can be inverted, leading to the conclusion that the vibration behaviour of samples of a particular material can be used to determine the material's elastic properties. Analytical formulas have been developed to calculate elastic moduli from the resonance frequencies of a beam-shaped test sample. European and American standard test procedures (ENV-843-2 and ASTM E 1876, respectively) propose, e.g. the following equation to calculate the E-modulus along the long axis of a beam-like sample from its flexural frequency

$$E = 0.946\left(\frac{mf_\mathrm{f}^2}{b}\right)\left(\frac{l}{h}\right)^3 A_\mathrm{f}$$

where E is Young's modulus, f_f is resonance frequency for fundamental mode flexural vibration, b is width of test piece, h is thickness of test piece, m is mass of test piece, and A_f is a shape factor.

This classical resonant beam test serves as an alternative to static tests in which a controlled load (or deformation) is applied while monitoring the resulting deformation (or load). The method that is most easy to interpret is tensile testing, which is used to determine the E-modulus of a long sample along its long axis from the slope of the uniaxial stress–uniaxial strain curve, obtained according to, e.g. ASTM E 111.

More recently, an inverse method, called 'Resonalyser procedure', was developed to determine all four engineering constants for orthotropic materials from the resonance frequencies of rectangular plate samples [2]. In this paper, standard tensile and resonant beam tests are used to validate the accuracy and sensitivity of the Resonalyser procedure for the case of rolled metal sheets.

2. Theoretical background to the Resonalyser procedure

2.1. The 'Poisson' test plate

The Resonalyser procedure is a mixed numerical-experimental method that aims to identify the engineering constants of orthotropic materials using measured resonance frequencies of freely suspended rectangular plates. Using rectangular plates as test specimen allows the simultaneous identification of E_1, E_2, ν_{12} and G_{12}. In addition, the obtained elastic material properties are homogenised over the plate surface and hence suitable as input values for finite element models of structures. Also the amount of machining induced edge damage is reduced when using plate shaped rather than elongated specimens.

The basic principle of the Resonalyser is to compare experimentally measured frequencies with the numerically computed frequencies of a finite element model of the test plate. Such an inverse procedure can only yield good results if the numerical model is controllable and if the elastic properties can be observed through the measured data [3,4]. This requires that in the selected series of frequencies at least one of the frequencies varies significantly for variations of each of the elastic properties. It can be shown that this requirement is fulfilled if the length to width ratio of the test plate approximately complies with

$$\frac{\text{Length}}{\text{Width}} = \sqrt[4]{\frac{E_1}{E_2}}.$$

A plate with such a ratio is called a Poisson test plate [5]. This name has been chosen based on the observation that the frequencies of the anticlastic and synclastic modes (Fig. 1) are particularly sensitive for changes of the Poisson's ratio of the plate material. A (hypothetical) material with a zero

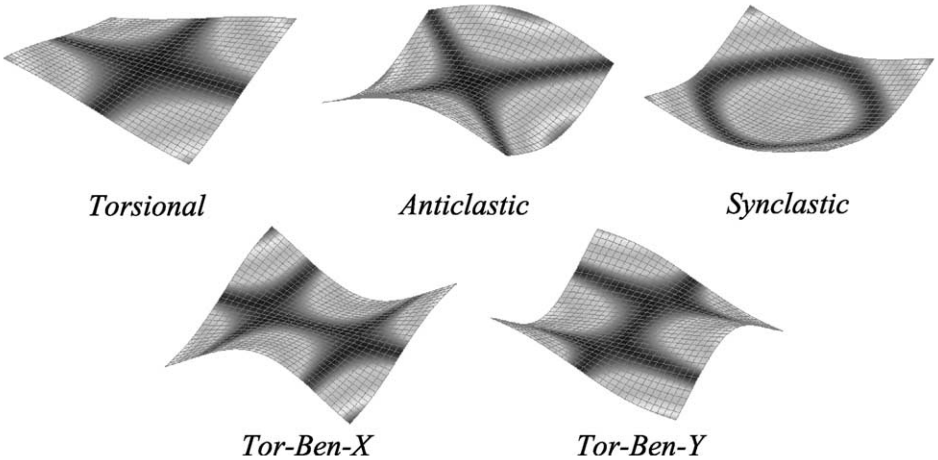


Fig. 1. The first five mode shapes of a Poisson test plate. The anticlastic mode is also called the 'saddle' mode. 'Breathing' of 'diaphragm' mode is commonly used equivalent terms for the synclastic mode.

value for Poisson's ratio would make the frequencies of both modes coincide.

Fig. 1 gives an overview of the five first mode shapes of a Poisson test plate. The mode shapes of the first three resonances (the torsional, anticlastic and synclastic modes) will always appear in this fixed sequence. The order of the fourth and fifth mode, an orthogonal couple of torsion-bending mode shapes, cannot be predicted a priori, and will have to be determined during the experiment.

2.2. Experimental modal analysis

To measure the five resonance frequencies, the test plate is suspended with thin wires in order to approximate the free–free boundary conditions of the FE-model as good as possible. The plate is excited by an impulse (a subtle hammer impact) and the vibrations are measured with a laservibrometer, which is connected with a standard PC equipped with a data acquisition card. This output only concept (the impulse force is not recorded) was chosen because the set-up can be easily used in a furnace, allowing use of the Resonalyser procedure to evaluate the temperature dependence of the material properties.

Traditionally, modal parameters are estimated from output only measurement data by means of the 'Peak Picking' method, which identifies the resonance frequencies from the position of the peaks in the output spectrum. Unfortunately, this simple approach does not allow the identification of closely spaced modes, and is therefore unable to separate the frequencies of the fourth and fifth mode of a Poisson plate. To identify these two torsion-bending modes, the Covariance-driven stochastic subspace identification method (SSI-Cov) was successfully used. Fig. 2 presents the stabilisation diagram of an aluminium plate as obtained with SSI-Cov. The overlay plot concentrates on the fourth and fifth mode, and the single peak in the spectrum shows that it is impossible to identify these two closely spaced modes by peak-picking. The two vertical lines of stable poles illustrate that the SSI-Cov method manages in separating the two torsion-bending modes. A full description of the SSI-Cov method can be found in Ref. [6].

2.3. Model updating: identification of the engineering constants

A detailed scheme of the Resonalyser procedure is given in Fig. 3. Starting with estimated initial values, the engineering constants in a finite element model of the test plate are iteratively updated until the first five computed resonance frequencies match the measured frequencies. In the finite element model, the plate dimensions and mass are considered as known quantities and thus fixed values. The four engineering constants are stored in the parameter column $\{p\}$. The updating of $\{p\}$ is realised by minimising a cost function $C(p)$

$$C(p) = \{f_{\mathrm{exp}} - f_{\mathrm{FEM}}(p)\}^{\mathrm{T}}[W^{(f)}]\{f_{\mathrm{exp}} - f_{\mathrm{FEM}}(p)\} + \{p^{(0)} - p\}^{\mathrm{T}}[W^{(p)}]\{p^{(0)} - p\} \quad (3)$$

in which $C(p)$ is a $\mathbb{R}^{\mathrm{NP}} \rightarrow \mathbb{R}$ cost function yielding a scalar value, $\mathrm{NP} = 4$ is the number of material parameters: E_1, E_2, ν_{12} and G_{12}, $\{p^{(0)}\}$ is a $(\mathrm{NP} \times 1)$ vector and contains the initial estimates for the material parameters, $\{f_{\mathrm{FEM}}(p)\}$ is a $(\mathrm{NF} \times 1)$ output column containing the $\mathrm{NF} = 5$ computed frequencies using parameter values $\{p\}$, $\{f_{\mathrm{exp}}\}$ contains the $(\mathrm{NF} \times 1)$ measured frequencies, $[W^{(f)}]$ is a $(\mathrm{NF} \times \mathrm{NF})$ weighting matrix applied on the difference between the measured and the calculated frequency column, $[W^{(p)}]$ is a $(\mathrm{NP} \times \mathrm{NP})$ weighting matrix for

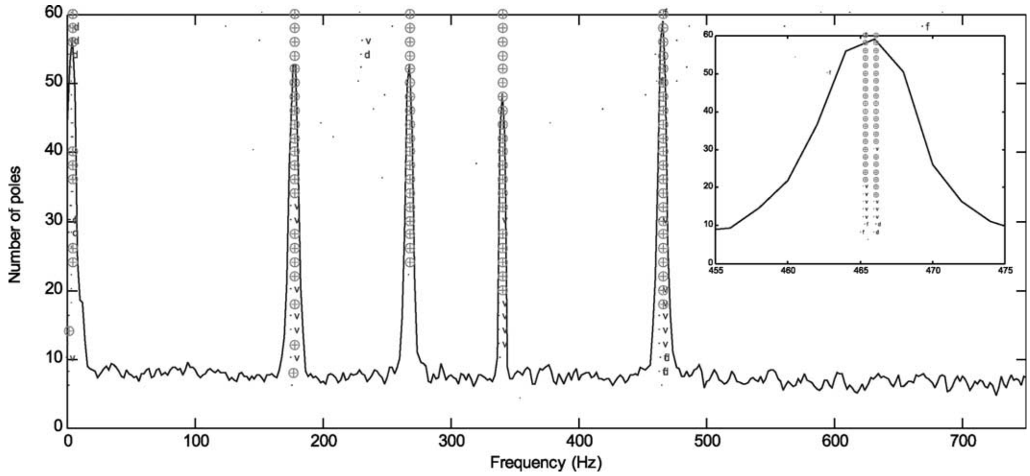


Fig. 2. The stabilisation diagram of an aluminium plate obtained with the SSI-Cov method. The used symbols are '⊕' for a stable pole, '.v' for a pole with stable frequency and vector, '.d' for a pole with stable frequency and damping, '.f' for a pole with stable frequency and '.' for a new pole, the solid line is the trace of the spectrum matrix.

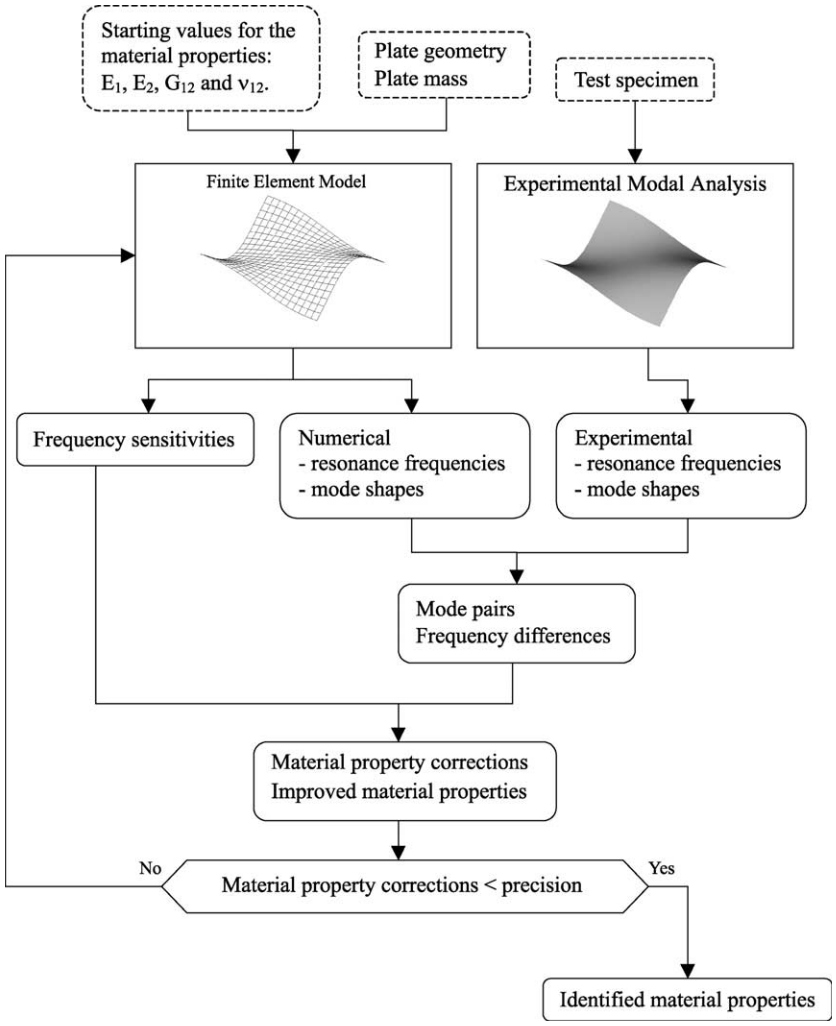


Fig. 3. Detailed flowchart of the Resonalyser procedure: material identification by comparing the experimentally measured and computed resonance frequencies of a test plate.

the difference between the initial parameter column $\{p^{(0)}\}$ and the parameter column $\{p\}$.

The cost function $C(p)$ has a minimal value for the optimal parameter values column $\{p^{(\mathrm{opt})}\}$. The value of this $\{p^{(\mathrm{opt})}\}$ can be made independent on the choice of the weighting matrices $[W^{(f)}]$ and $[W^{(p)}]$ in the cost function. The choice and role of $[W^{(f)}]$ and $[W^{(p)}]$ is discussed, among others, in Refs. [1,3,7]. The updating of the initial parameter column toward $\{p^{(\mathrm{opt})}\}$ by minimisation of the cost function is given by the following recurrence formula in iteration step $(j+1)$:

$$\{p^{(j+1)}\} = \{p^{(j)}\} + [W^{(p)} + S^{(j)^{\mathrm{T}}} W^{(f)} S^{(j)}]^{-1} S^{(j)} W^{(f)} \times \{f_{\mathrm{exp}} - f_{\mathrm{FEM}}(p^{(j)})\} \tag{4}$$

In Eq. (4) S is the sensitivity matrix containing the partial derivatives of the numerical frequencies with respect to the elements of the parameter column.

The numerical model of the test plate is based on the Love–Kirchhoff theory [1]. The applicability of this theory is mainly limited by the thickness of the plate. Traditionally, plates with a length/thickness ratio that exceeds a factor of 50 are considered as sufficiently thin. The tested materials and applied vibration amplitudes do not violate additional assumptions made by the Love–Kirchhoff theory. Very accurate eight order polynomial Lagrange functions are taken as shape functions in the used numerical finite element model of the test plate [8]. The stiffness matrix of the test plate is evaluated in each iteration cycle using standard finite element procedures with the values of the parameter column $\{p\}$ at that moment [9]. The computed resonance frequencies are obtained by solving a generalised eigenvalue problem composed with the constant mass matrix and the evaluated stiffness matrix [10]. The iteration procedure ends if convergence of $\{p\}$ is reached. The values of the engineering constants in $\{p\}$ after the last iteration cycle are considered as the result of the Resonalyser procedure.

The whole identification procedure takes typically less than 5 s on a standard pentium-III PC.

3. Material identification

3.1. Test specimen selection and preparation

Commercially available 6082 Al-alloy and 304 stainless steel plates were obtained in the hot-rolled state. The 304 stainless steel is an austenitic steel produced by Avesta AB. Main alloying elements are chromium (18.5%) and Ni (8.5%). The plate used in this investigation has a thickness of 6 mm. The 6082 aluminium alloy contains as main alloying elements silicon (0.7–1.3%), iron (0.5%), Mn (0.4–1.0%) and Mg (0.6–1.2%). The plate used in this investigation has a thickness of 5 mm.

Relatively large tensile (200×20 mm^2), resonant beam (300×24 mm^2) and Resonalyser (300×300 mm^2) test samples were produced from a single plate. Length and width of the samples were carefully machined (to within ± 0.03 mm) to eliminate or at least reduce the effect of inaccurate sample dimensions on the calculated material properties. The thickness of the samples was not changed: the rolling surfaces were left untouched. The standard deviation of the thickness was 0.013 mm. Resonant beam and tensile test samples were cut along the rolling direction and at the following angles: −90, −60, −45, −30, 0, 30, 45, 60 and 90°. The plates for the Resonalyser tests had sides parallel to and at 90° with the rolling direction.

3.2. Standard mechanical tests

The elastic moduli were determined following standard resonant beam and tensile test procedures. The resonant beam method is based on the measurement of the fundamental flexure and torsion resonance frequencies of slender beam samples. Analytical equations, based on elastic beam theory, relate these frequencies to the Young's and shear modulus while assuming isotropic material behaviour. Resonant beam tests were performed on rectangular beam-like samples using the RFDA apparatus (IMCE nv, Diepenbeek, Belgium) which analyses the resonant vibration obtained with a mechanical or acoustic impulse excitation [11,12], following the guidelines provided by ASTM E 1876-99 and ENV-843-2.

Tensile tests were performed on flat dog-bone shaped samples of length 200 mm, 65 mm gauge length and gauge sections of 63 mm^2 for the aluminium and 73 mm^2 for the steel samples. The strain was determined in a first instance using a clip-on extensometer (gauge length 50 mm). Afterwards, tensile tests were repeated on the same samples, now using bi-axial strain gauges. For each test, the load was applied and removed periodically, with an increasing amplitude (3, 6 and 9 kN for the aluminium samples, and 2, 4 and 6 kN for the steel samples).

4. Experimental results

4.1. Resonant beam tests

The Young's modulus was calculated from the fundamental in plane bending frequency (IP-Bending). The shear

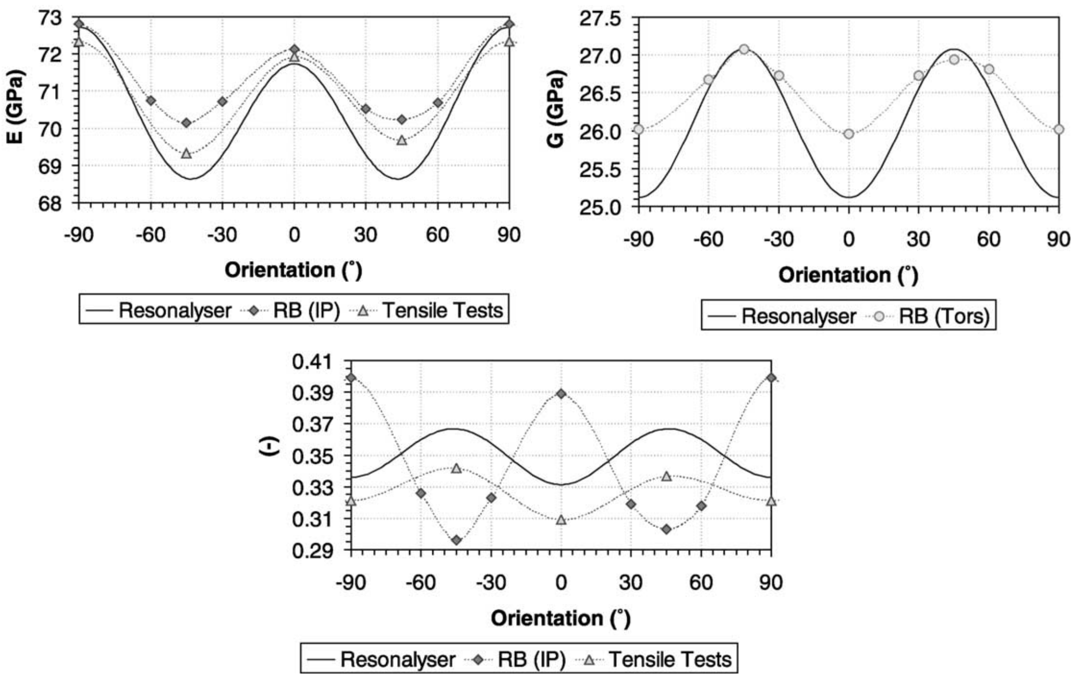


Fig. 4. Comparison of the results of the three different methods for the E-, G-modulus and Poisson's ratio ν for aluminium.

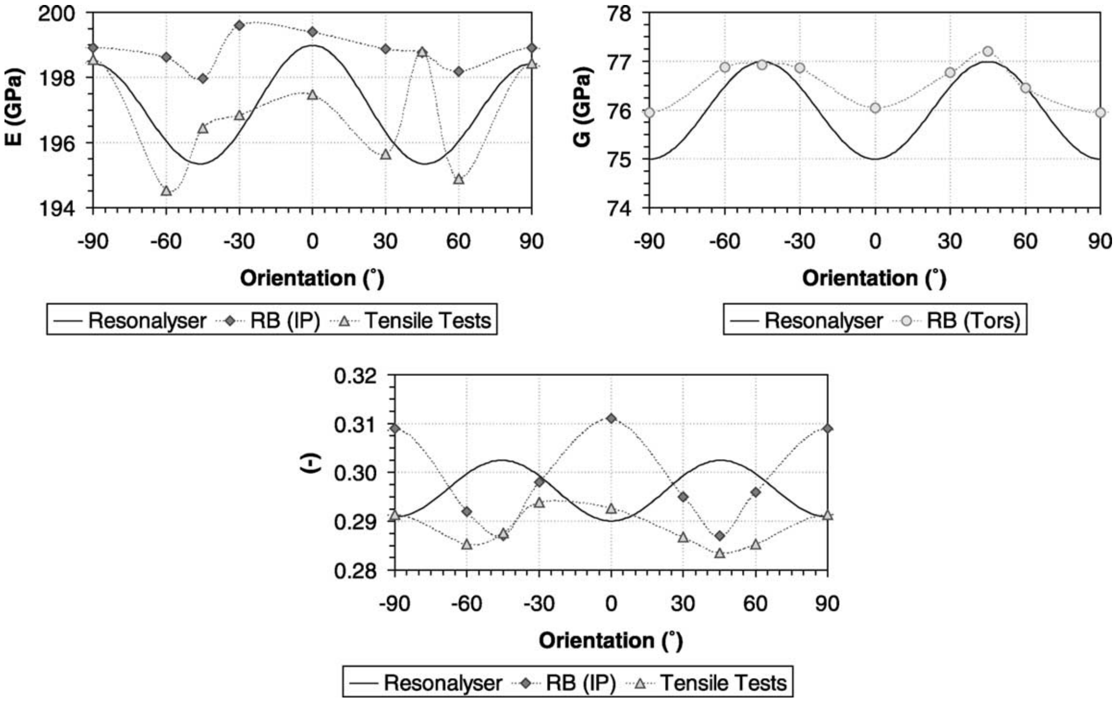


Fig. 5. Comparison of the results of the three different methods for the E-, G-modulus and Poisson's ratio ν for steel.

modulus was calculated from the fundamental torsion frequency of the beam. Figs. 4 and 5 show the obtained material properties for different orientations of the test beams.

4.2. Uniaxial tensile tests

The Poisson's ratios were identified by means of bi-axial strain gauges, while the measurement of the Young's moduli was performed with a clip-on extensometer. The values of the elastic moduli were derived from the stress–strain curve obtained during the loading phase of the test cycle. The measured material properties are plotted in Figs. 4 and 5.

4.3. Resonalyser tests

The material properties were obtained with the procedure described in Section 2. Tables 1 and 2 compare the measured resonance frequencies with the analytical frequencies of the Resonalyser's FE-model.

The Resonalyser identifies the material properties in the direction of the main sample axes. The off-axis elastic properties of an orthotropic material can be calculated with the following equations [13]

$$\frac{1}{E_x}=\frac{1}{E_1}\cos^4\theta+\left(\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_1}\right)\sin^2\theta\cos^2\theta+\frac{1}{E_2}\sin^4\theta$$
$$\nu_{xy}=E_x\left[\frac{\nu_{12}}{E_1}(\sin^4\theta+\cos^4\theta)-\left(\frac{1}{E_1}+\frac{1}{E_2}-\frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta\right]$$
$$\frac{1}{E_y}=\frac{1}{E_1}\sin^4\theta+\left(\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_1}\right)\sin^2\theta\cos^2\theta+\frac{1}{E_2}\cos^4\theta$$
$$\frac{1}{G_{xy}}=2\left(\frac{2}{E_1}+\frac{2}{E_2}+\frac{4\nu_{12}}{E_1}-\frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta+\frac{1}{G_{12}}(\sin^4\theta+\cos^4\theta) \quad (5)$$

in which θ is the angle between the main direction and the considered off-axis direction.

Table 1
The obtained frequency match for the aluminium plate

	Freq. exp. (Hz)	Freq. num. (Hz)	Difference (%)
Mode 1	177.00	176.76	0.13
Mode 2	267.50	267.36	0.05
Mode 3	339.94	339.79	0.04
Mode 4	465.35	465.88	−0.11
Mode 5	466.09	466.62	−0.11

Table 2
The obtained frequency match for the steel plate

	Freq. exp. (Hz)	Freq. num. (Hz)	Difference (%)
Mode 1	220.53	220.09	0.20
Mode 2	322.73	322.51	0.07
Mode 3	400.92	400.67	0.06
Mode 4	562.86	563.82	−0.17
Mode 5	576.96	577.86	−0.16

4.4. Comparison of the results

As opposed to the Resonalyser results, the off-axis properties identified with resonant beam and tensile tests are directly measured on samples cut from the plate at an angle θ with the rolling direction. Figs. 4 and 5 compare these results with the curves obtained with the formulas of Eq. (5) and the Resonalyser procedure. The anisotropy of the Al-material is clearly more pronounced (about 4% difference between minimum and maximum Young's modulus) than that of the investigated stainless steel (less than 2%).

5. Discussion

A comparison of the results of the Resonalyser technique with the resonant beam (RB) test results leads to a number of peculiar observations:

- similar results are found for the Young's modulus in the two main directions 0° and $\pm 90°$;
- there is a difference between the Young's moduli for the off-axis directions: $E_{RB} > E_{resonalyser}$;
- there is a difference between the values of the shear modulus at 0°: $G_{RB} > G_{resonalyser}$;
- there is no agreement between the results for Poisson's ratio.

5.1. The influence of warping

The Kirchhoff theory assumes that a plane, perpendicular to the central plane of the plate before deformation, remains flat and perpendicular to the central plane after deformation. The Kirchhoff thin plate theory does not account for warping deformations caused by shear stresses induced by torsion. For a given set of material properties, this leads to an overestimation of the resonance frequency of all the torsion modes. The Resonalyser procedure will compensate this effect by artificially reducing the shear modulus. This is exactly what is observed when comparing the resonant beam and Resonalyser results at 0°.

The problem could be overcome by using a more complex FE-model, such as a 3D model, which does not neglect warping. But the use of such a model requires a number of additional material parameters: E_3, G_{13}, G_{23}, ν_{13} and ν_{23} (1). Since all these parameters are unknown and independent, they would have to be identified, and this would drastically increase the complexity of the identification procedure. Because the Kirchhoff model especially overestimates the frequencies of the modes with a torsional deformation, a more practical approach would be to identify the in-plane elastic properties from a number of pure bending frequencies only.

As already stated in Section 2.1, an inverse method can only yield good results if the elastic properties can be observed through the measured data [2,3]. The bending modes of the used test plate, a plate of which the edges are parallel with the main material directions, are very insensitive to a variation of the shear modulus. The identification of the shear modulus without using torsional mode shapes thus requires a second test-plate, of which the bending frequencies are highly shear modulus sensitive. A good solution to ensure a maximal sensitivity with respect to the shear modulus, is the use of a test plate of which the diagonals are parallel with the two main directions. The frequencies of the bending modes of such a plate will be controlled by the 45° off-axis Young's modulus, which is very sensitive to a variation of the in-plane shear modulus.

Table 3
The obtained frequency match for the aluminium plates

	Freq. exp. (Hz)	Freq. num. (Hz)	Difference (%)
Mode 1	267.50	267.55	0.02
Mode 2	339.94	339.25	−0.20
Mode 3	844.20	844.36	0.02
Mode 4	849.38	849.59	0.02
Mode 5	260.64	260.56	−0.03
Mode 6	335.17	335.75	0.17

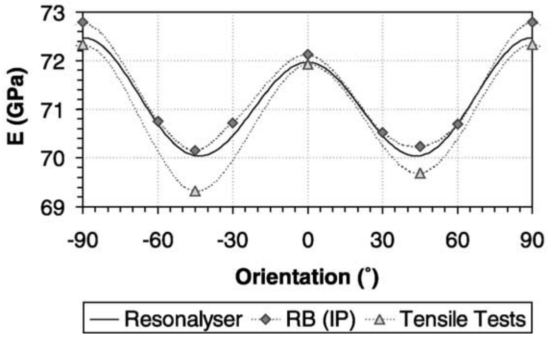
An aluminium plate was prepared with edges at 45° to the rolling direction. Table 3 shows the obtained frequency match between the measured and the numerical frequencies for the adapted Resonalyser procedure. The adapted procedure is based on the same framework as the standard procedure, but uses a different set of frequencies. Table 4 presents the modes that are used in the adapted procedure.

Fig. 6 shows that the adapted Resonalyser procedure does not only remove the discrepancy between the resonant beam and Resonalyser shear modulus, it also solves the underestimation of the off-axis properties of the Resonalyser.

For very thin plates, the mixed numerical-experimental Resonalyser technique can identify the four elastic properties of an orthotropic material from the first five resonance frequencies of a single test plate. When the thickness of the plates increases, the application of the original Resonalyser procedure will result in an underestimation of the shear modulus, because the Kirchhoff plate theory, which is used in the Resonalysers FE-model, cannot model

Table 4
The bending modes used in the adapted Resonalyser procedure

	Mode type	Plate	Sensitive to
Mode 1	Saddle	0–90°	ν_{12}
Mode 2	Breath	0–90°	ν_{12}
Mode 3	Bending 2-Y	0–90°	E_2
Mode 4	Bending 2-X	0–90°	E_1
Mode 5	Saddle	−45–45°	G_{12}
Mode 6	Breath	−45–45°	G_{12}



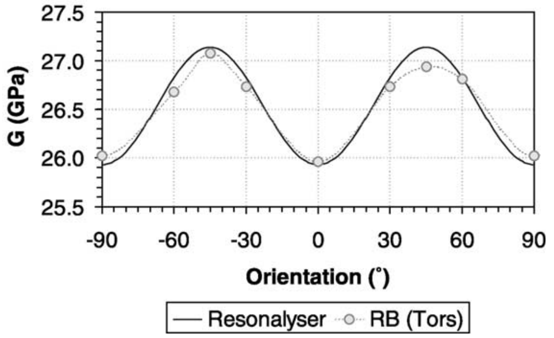


Fig. 6. The squares and triangles represent the material properties measured with resonant beam and tensile tests, respectively, the curves show the results of the adapted Resonalyser procedure.

the influence of warping. To identify the correct material properties an adapted Resonalyser procedure that uses six vibration frequencies of two test plates will have to be applied. To identify the physically correct values of the elastic material properties the adapted procedure is preferred. However, when the material properties have to be used as input data for 2D finite element models based on the Kirchhoff theory, the original procedure is preferred, since the obtained material properties will compensate the similar shortcomings of the FE-model, and their use will result in a better correlation between FE calculation and experimental results.

5.2. Comparison of Poisson's ratio

The comparison between the Poisson's ratio obtained with resonant beam and (adapted) Resonalyser reveals a complete lack of agreement between the results of these two methods. However, the tensile tests and the Resonalyser procedure reveal the same directional dependency of Poisson's ratio. Fig. 7 shows a high similarity between the directional dependency of the elastic and shear modulus measured with resonant beam and tensile tests, and the directional dependency calculated with Eq. (5) using the four main material properties as identified with the adapted Resonalyser procedure. However, when the curves are calculated using the same values for E_1, E_2 and G_{12}, but taking the Poisson's ratio as identified with resonant beam tests, they completely fail to model the direction variation of both elastic and shear modulus. It is clear that resonant beam tests fail to identify Poisson's ratio for orthotropic materials. The reason for this is the fact that the resonant beam-identification-procedure is based on a relation between Poisson's ratio and the elastic and shear modulus, which only holds for fully isotropic materials.

5.3. Additional remarks

For the highly homogeneous materials that were used in this investigation, a good repeatability was obtained for both the Resonalyser and resonant beam tests. For materials with less homogeneous elastic properties, like composite materials, the Resonalyser results will be less sensitive to local differences, since the technique identifies the average stiffness of a large plate specimen. Therefore, good average values of the elastic materials properties can be obtained with less experiments than with resonant beam tests. Obviously, the use of the Resonalyser technique is not recommended when the homogeneity of the material has to be investigated.

The repeatability of the results was much lower in the case of tensile tests, than in the case of the resonant vibration test methods (resonant beam and Resonalyser). The uncertainty on the results obtained with tensile tests is too large to validate the absolute values of the material properties obtained with Resonalyser technique. However, the results of the tensile tests do confirm the directional dependency of the material properties as measured with the Resonalyser, at least for the Al alloy. The elastic anisotropy of the hot-rolled stainless steel is too small (about 1%) to be more than detected with any of the test methods used.

The full potential of the Resonalyser technique has not been used yet. The mixed numerical–experimental techniques are, theoretically, capable of measuring any property that can be used as an input parameter for a FE-model. One of the possible applications of mixed numerical–experimental techniques currently investigated,

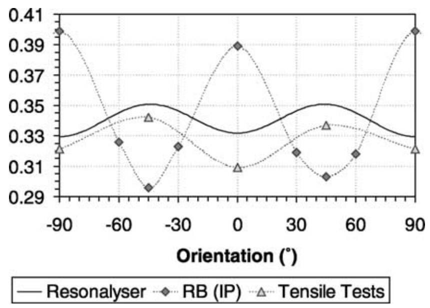


Fig. 7. Comparison of Poisson's ratio of the investigated Al, obtained with the adapted Resonalyser procedure, tensile testing and calculated from resonant beam tests.

is the identification of the elastic properties of the individual layers of a layered material.

6. Conclusions

1. In this paper three methods for measuring the elastic properties of plate materials are shown to quantify the degree of elastic anisotropy of aluminium and stainless steel sheets. The results obtained with different techniques (uniaxial tensile tests, resonant beam tests and the Resonalyser procedure, a mixed-numerical–experimental technique based on impulse excited resonant vibrations of Plate) confirm each other.
2. Although both resonant beam and Resonalyser tests deliver the same results for Young's and shear modulus, only the Resonalyser procedure is able to obtain correct results for the Poisson's ratio of orthotropic materials.
3. Both resonant beam and Resonalyser procedures manage to accurately determine the directional dependence of in-plane stiffness, even for a moderately anisotropic hot-rolled Al alloy, for which minimum and maximum stiffness are $<4\%$ different.

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